Int. J. Heat Mass Transfer. Vol. 10, pp. 1195–1210. Pergamon Press Ltd. 1967. Printed in Great Britain

THERMAL AND MATERIAL TRANSPORT FROM SPHERES

PREDICTION OF MACROSCOPIC THERMAL AND MATERIAL TRANSPORT

T. R. GALLOWAY and B. H. SAGE
Chemical Engineering Laboratory, California Institute of Technology, Pasadena, California

(*Received* 24 *November* 1966)

Abstract—From a review of the available information concerning the effect of the molecular properties of the fluid, conditions of flow, and level of turbulence, a correlation of the Frössling number for the macroscopic or space average transport from spheres has been prepared. The effect of level of turbulence is most marked at a Reynolds number of 40000, and becomes relatively unimportant at Reynolds numbers above 250000. Analytical expressions, empirical in nature, were developed to describe the influence in Reynolds number, levels of turbulence, and the molecular properties of the fluid upon the Frössling number. The accuracy of prediction of the Frössling number is probably comparable to the accuracy of the experimental data. The ratio of the kinematic viscosity of the free stream to that at the interface raised to a power has been found a useful, although empirical, method of correcting for the changes in the molecular properties throughout the boundary flows. The results are presented in graphical and tabular form.

NOMENCLATURE

A, B, C, D, E, F, G, H, coefficients;
b, specific gas constant [psi ft^3/lb °R];
C_p, isobaric heat capacity [Btu/lb °F];
D_C, Chapman–Cowling diffusion coefficient [ft^2/s];
D_M, Maxwell diffusion coefficient [lb/s];
d, diameter [in or ft];
Fs, Frössling number, $(Nu - 2)$ or $(Sh - 2)/Re^{\frac{1}{2}}Pr_m^{\frac{1}{3}}$ or $Sc_m^{\frac{1}{3}}$;
f, fugacity [lb/ft^2];
h, heat-transfer coefficient [Btu/s ft^2 °F];
k, mass-transfer coefficient [ft/s];
k, thermal conductivity [Btu/s ft °F];
ln, natural logarithm;
$\dot{m}$, evaporation rate [lb/s ft^2];
N_c, number of constants;
N_p, number of data points;
Nu, Nusselt number (hd/k);
n, mole fraction;
p, pressure [lb/in^2 abs. or lb/ft^2 abs.];
Pr, Prandtl number (κ/ν);
Re, Reynolds number (dU/ν);
Sc, Schmidt number, $\nu/D_{Ck_j} = P\nu/D_{Mk}$;
Sh, Sherwood number, $kd/D_{Ck_j} = \dot{m}_k b_k T/D_{Mk}Z(f_k^\circ/P)\ln(n_{ji}/n_{j\infty})$;
s, average deviation, fraction (defined in Tables 2 and 3);
T, absolute temperature [°R];
U_∞, free-stream velocity [ft/s];
u, local velocity [ft/s];
$\bar{u}'_{zf}$, mean longitudinal fluctuating velocity [ft/s];
w, weighting factor, $1/100\sigma$;
z, compressibility factor.

Greek symbols

α_t, apparent level of turbulence, fraction, $[(\bar{u}'_{zf})^2]^{\frac{1}{2}}/U_\infty$;
K, thermometric conductivity [ft²/s];
ν, kinematic viscosity [ft²/s];
Σ, summation operator;
σ, specific weight [lb/ft³];
σ, standard deviation, fraction (defined in Tables 2 and 3);
ψ, angle from stagnation.

Subscripts

c, calculated;
e, experimental;
i, evaluated for interface conditions;
j, component j;
k, component k;
m, molecular property;
∞, evaluated for free stream conditions.

Superscripts

m, n, exponents;
$*$, macroscopic or surface average;
0, pure component.

INTRODUCTION

DURING the past decade there has been marked increase in interest in the local transport from spheres and cylinders to turbulent streams. The more recent experimental work has revealed the complicated nature of such transport phenomena. A review and limited analysis of the development in this field stressing the more recent literature is available [1]. The early work of Morse [2], Langmuir [3], Fuchs [4], and Frössling [5], as well as that of Ranz and Marshall [6] and others [7–10], gives indication of the background available. A few authors treated the effects of free-stream turbulence [11–16], and others the regions of supercritical flow [17–21]. The effects of the Prandtl and Schmidt numbers, free convection, variable molecular properties of the fluid, turbulence level, and behavior in the wake were considered [22–62], but few definite conclusions were drawn. There resulted a collection of experimental data which was extensive and, in some cases, repetitive but often conflicting.

The available experimental data have been incorporated into a method of prediction wherein the transport is considered to be a function of the Reynolds number, the Prandtl or Schmidt number, and the level and scale of turbulence. The results are presented in analytical form, and statistical measures of the deviation from the experimental information available are presented in appropriate tabulations. The available experimental data for 0·5 and 1·0-in spheres involving only air [13–15, 24, 25] do not permit the consideration of the variation of Prandtl and Schmidt numbers on the local boundary flows. Insufficient experimental data appear to be available for fluids other than air to permit quantitative evaluation of the effect of the Prandtl and Schmidt numbers. The small amount of data available indicates that the $Pr^{\frac{1}{3}}$ and $Sc^{\frac{1}{3}}$ are suitable semiquantitative means of evaluating the influence of the molecular properties of the free stream. Throughout this discussion, the authors have used the phrase "macroscopic" transport as equivalent to the surface average value.

ANALYSIS

As a first-order approximation, it may be assumed that the macroscopic Nusselt number for forced convection may be related to the Reynolds number by the following quasitheoretical expression first developed by Frössling [5]:

$$Nu^* \cong 2 + 0{\cdot}552\, Re^{\frac{1}{2}}\, Pr^{\frac{1}{3}}. \qquad (1)$$

At low Reynolds numbers, the data indicate the limiting value of the Nusselt number to be substantially 2 as shown by the early measurements of Morse [2] and set forth theoretically by Langmuir [3]. Equation (1) is based on solutions of equations of motion describing the flow in the vicinity of the stagnation point. Transport from a sphere involving laminar boundary flows at low free-stream turbulence levels is described satisfactorily by this relation [5]. Quantitative theoretical calculations at the stagnation point can be made, using both exact solutions for potential flow distributions and approximate integral solutions for various flow distributions established by experiment. Korobkin [63] reviewed these techniques, and the pertinent expression for the local thermal transport at the stagnation point was:

$$Nu = 2 + 1{\cdot}31\, Re^{\frac{1}{2}}\, Pr^{\frac{1}{3}}. \tag{2}$$

As the flow proceeds around the sphere, separation of the boundary layer occurs, resulting in a gross alteration of the mechanism of thermal and material transport. Richardson [50, 51] suggested that this region of flow be considered of such characteristic that the transport is dependent to a greater extent upon the Reynolds number than in laminar boundary flows. This dependence on Reynolds number is reflected in the experimental data from a number of studies [8, 17, 18, 20, 34, 46, 52, 57, 64].

Many efforts have been made to represent the macroscopic or surface average subcritical transport from spheres by the following empirical expressions:

$$Nu^* = A\, Re^m\, Pr^n, \tag{3}$$

$$Sh^* = A\, Re^m\, Sc^n. \tag{4}$$

Several authors [28, 29, 33, 35, 58] have reviewed the trends in the coefficients in such empirical expressions. Such relations are not satisfactory since the exponents are not single-valued. As a result of this deficiency, it was desirable to investigate the causes for failure of relations of the form of equation (1) to describe the transport from spheres.

The quantity

$$Fs = \frac{Nu - 2}{Re^{\frac{1}{2}}\, Pr^{\frac{1}{3}}} \tag{5}$$

will be referred to as the Frössling number, Fs, for spheres following the recent proposal for flat plates [41]. The Frössling number defined by equation (5) is a useful parameter to describe the behavior in the boundary flows as a result of variations in the level of turbulence, together with the inception of separation. For material transport the analogous number becomes:

$$Fs = \frac{Sh - 2}{Re^{\frac{1}{2}}\, Sc^{\frac{1}{3}}}. \tag{6}$$

Actually, the Frössling number of equation (5) and the number defined by equation (6) are not identical. However, within the uncertainties of the measurements available, they appear numerically equal, and they have been given the same symbol in all subsequent equations. The defects in such an analogy are discussed later.

In arriving at a more satisfactory relation to describe macroscopic transport, the factors influencing the local transport will be considered first. These local effects are then combined into a single expression for the macroscopic or surface average transport. There are several physical factors which result in a variation of the Frössling number with conditions of flow. The more important of these factors appear to be the greater dependence of the transport upon the Reynolds number in the region of separated flow, free-stream turbulent perturbation of the laminar boundary flows, migration of the locus of separation, influence of the integral scale of the free-stream turbulence, and the transition to a turbulent boundary layer upstream of the point of separation which is encountered in supercritical flows. The transition of the laminar boundary flow to turbulent motion upstream of separation is commonly called "supercritical flow" [47, 65].

In Fig. 1 the data indicate the limitation of the following empirical expression for local transport, which is based in part on equation (1) for macroscopic transport from spheres:

$$Nu = 2 + C(\psi)\, Re^m\, Pr^n. \tag{7}$$

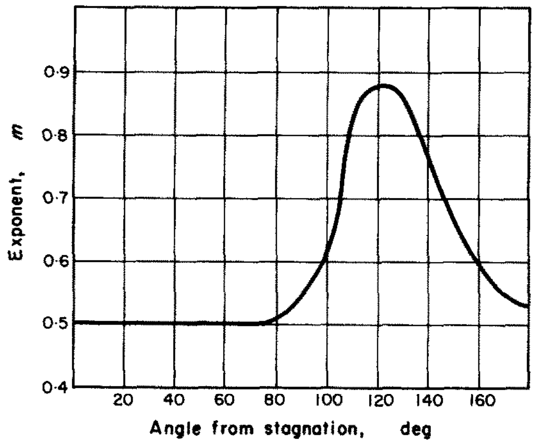

FIG. 1. Influence of position upon exponent in equation (7).

The marked variation with polar angle in the exponent, m, of the Reynolds number is evident.

The effects of variations in the free-stream turbulence level have been shown [66] to increase the transport through the laminar boundary layer in the forward hemisphere. The influence of the level of turbulence apparently decreases with an increase in thickness of the boundary flows. There is a decrease in the absolute influence of the turbulence level with the increase in the polar angle from stagnation [13]. However, the relative effect of turbulence increases with an increase in the polar angle. The summation is illustrated in Fig. 2 for macroscopic or surface average thermal transport [13–16, 24, 25].

Utilizing the random eddy penetration model [40] for the interaction of turbulent perturbations with the laminar boundary flow, the variation of the Frössling number may be described by the following expression for the forward hemisphere and laminar portions of the wake:

$$Fs_{\infty} = A + B\,\alpha_t(\alpha_t + C)\,Re_{\infty}^{\frac{1}{2}}\,Pr_{m,\infty}^{\frac{1}{3}}. \quad (8)$$

This relation was obtained by ascribing a local velocity and temperature perturbation to the eddy conductivity or diffusivity. Equation (8) yields an augmentation of the transport in the laminar region. Such behavior also was predicted qualitatively by Harriott [40] using a stochastic approach to the eddy penetration theory.

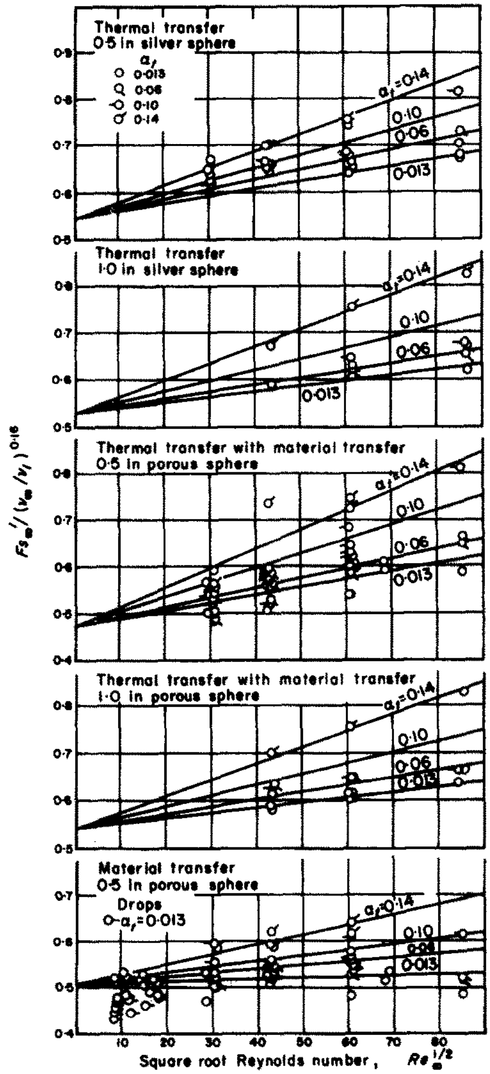

FIG. 2. Macroscopic Frössling number for several levels of turbulence.

In the turbulent, separated-flow region of the wake, the local Frössling number can be characterized as independent of the level of free-stream turbulence and may be described by:

$$Fs_{\infty} = E + F\,Re_{\infty}^{\frac{1}{2}}\,Pr_{m,\infty}^{m}. \tag{9}$$

Two distinct mechanisms of transport were recognized in the wake, that characterized by a laminar and that associated with a turbulent boundary flow. Near 108° from stagnation the laminar layer separated, and the region downstream was treated as turbulent. In the turbulent wake, vortex formation may result in the creation of a laminar back flow. Thus some laminar regions in the wake may be described by equation (8), while the turbulent portions are represented by equation (9).

When the effects of the wake and the free-stream turbulence are treated as being independent, equations (8) and (9) can be combined in the following form to describe the effect of Reynolds number on the macroscopic or space average transport from the sphere in the subcritical regions:

$$Fs_{\infty}^{*} = A^{*} + [B^{*}\alpha_t(\alpha_t + C^{*}) + D^{*}]\,Re_{\infty}^{\frac{1}{2}}\,Pr_{m,\infty}^{m}. \tag{10}$$

The following analogous expression for material transport results and the coefficients are numerically equal to those in equation (10):

$$Fs_{\infty}^{*} = A^{*} + [B^{*}\alpha_t(\alpha_t + C^{*}) + D^{*}]\,Re_{\infty}^{\frac{1}{2}}\,Sc_{m,\infty}^{m}. \tag{11}$$

The asterisk was chosen to denote the surface average of the local transport over the surface of the sphere.

Equations (10) and (11), with appropriate numerical coefficients for the subcritical region of flow, are compared with the experimental data available [7, 11, 18, 21, 27, 30, 31, 35, 46, 57, 58] for thermal and material transfer from spheres in Fig. 3. The upper diagram shows the abundance of data at low Reynolds numbers while the rather complicated behavior in the transition region is depicted in the lower part of Fig. 3.

The effects of scale of turbulence are shown in Fig. 4. It can be seen from the trends depicted that although Maisel and Sherwood [11] did vary the integral scale of turbulence, the maximum scale was smaller than the diameter of the sphere. Figure 4 includes data [13–15, 24, 25] involving an integral scale of turbulence comparable to the diameter of the sphere employed. Some trends for flow about a cylinder reported by Van der Hegge Zijnen [59] are included for comparison in Fig. 4.

The transition of the laminar boundary flow into a turbulent regime before separation is shown by the simple dependence of the Frössling number upon the Reynolds number, as depicted in Fig. 3 for the higher Reynolds number. The influence of turbulence intensity on the transition Reynolds number for transport was estimated from the available data for thermal transport and is depicted in Fig. 5. For comparison, the transition Reynolds number obtained from the drag measurements of Dryden [67] is included. An empirical representation of the Frössling numbers in supercritical flow at Reynolds numbers above 360000 results:

$$Fs_{\infty}^{*} = G^{*} + H^{*}\,Re_{\infty}^{\frac{1}{2}}\,Pr_{m,\infty}^{m}. \tag{12}$$

The variation of physical properties through the boundary layer should be accounted for by using only molecular properties. A form of solution for subcritical flow allowing for variable molecular properties might utilize the ratio of kinematic viscosities of the free stream and at interfacial conditions,

$$Fs_{\infty}^{*} = A^{*}\left(\frac{\nu_{\infty}}{\nu_i}\right)^{n} + [B^{*}\alpha_t(a_t + C^{*}) + D^{*}]\,Re_{\infty}^{\frac{1}{2}}\,Pr_{m,\infty}^{m} \tag{13}$$

with all other molecular and flow properties evaluated at the state of the free stream. The validity of this approach can be seen in Fig. 6 for some recent experimental data [13–15, 24, 25] utilizing air and the data of Vliet [30] for water. The value of the exponent, n, of 0·16 was established from all of the data for air [13–15, 24, 25] and water [30]. Such a procedure, when data

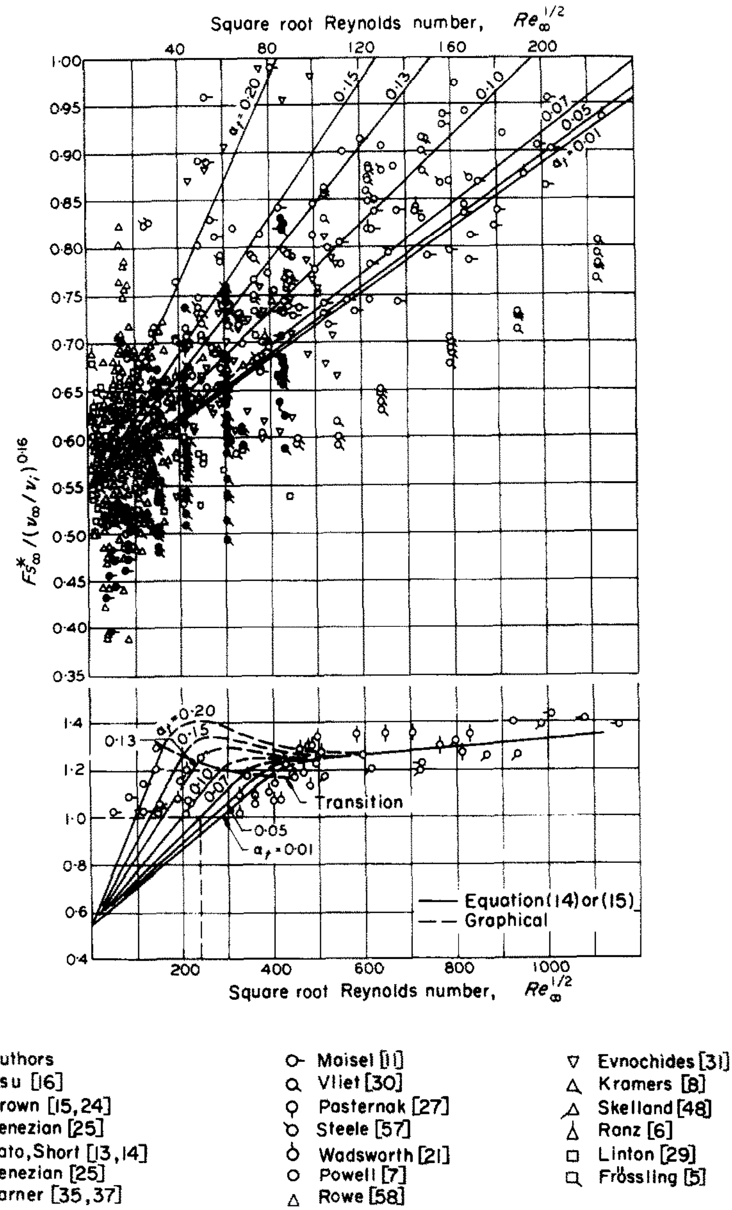

FIG. 3. Macroscopic Frössling number from several investigators.

at all Reynolds numbers and levels of turbulence are considered in a regression analysis, yields a flat minimum at a value of the exponent of 0·16. The full curves are based on equation (13) while the points are experimental values. The data for water are higher, owing to the increased non-linear wake effect encountered at the higher molecular Prandtl numbers. Without a detailed knowledge of the variation of the properties throughout the boundary layer, the use of effective molecular properties is, at best, only a fair approximation.

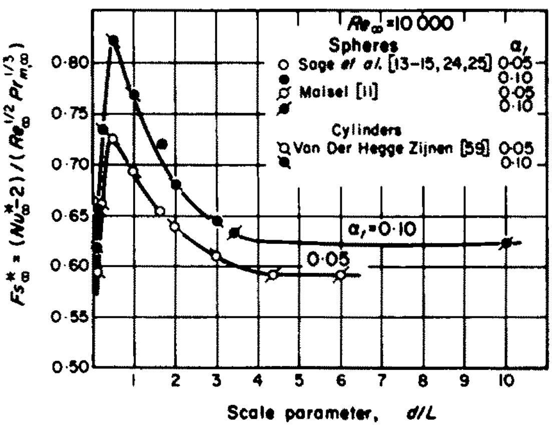

FIG. 4. Macroscopic Frössling number as function of conditions of flow.

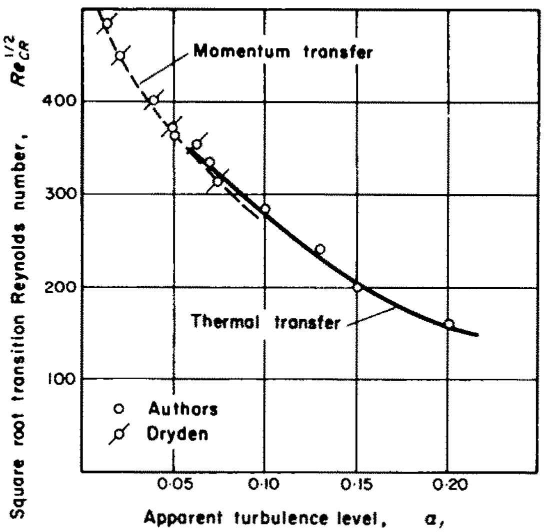

FIG. 5. Influence of level of turbulence upon transition Reynolds number.

For Reynolds numbers greater than 400, natural convective transport is usually negligible compared to forced convection. The magnitude of natural convection becomes more pronounced for large temperature or concentration gradients. At Reynolds numbers well below 400, the combined interaction of free and forced convection has been shown by Klyachko [22] and Yuge [23] to result in a minimum in the Nusselt number. It is desirable to recognize effects of local surface temperature variation in

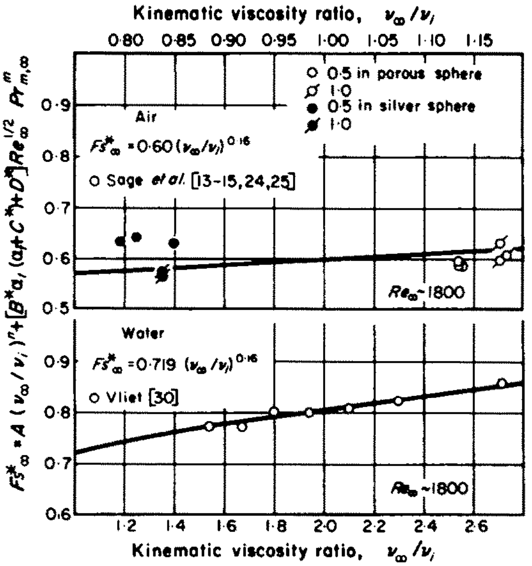

FIG. 6. Effect of kinematic viscosity ratio upon macroscopic Frössling number.

evaluating the effective driving force. An often neglected effect is a finite momentum velocity normal to the interface associated with material transport. The "blowing velocity" yields smaller temperature and concentration gradients by increasing the boundary flow thickness, thus decreasing the thermal or material transfer coefficient. Such effects for spheres have been estimated by analytical methods [60, 61]. Taking into account the error in an earlier paper [60], the effect of the "blowing boundary layer" on material transport [13, 15, 24, 25] would decrease the Frössling number by 7 per cent as shown in Fig. 7 for a "blowing parameter", M, of approximately 0·1. The single experimental point represents the average of data obtained by the authors for 0·5- and 1·0-in silver and porous spheres [13, 15, 24, 25]. An effect of this magnitude has also occurred in the work of other investigators [7, 11, 28, 29, 31–40, 48, 49].

Several investigations involving transport from spheres have been carried out in wind tunnels [17–21]. When the area of the sphere cross section is comparable to the area of the tunnel working section, the average velocity

Table 1. Range of

Description	Number of points	Turbulence level min.	Turbulence level max.	Prandtl number min.	Prandtl number max.	Schmidt number min.	Schmidt number max.
n-heptane drops	33	0·013				2·130	
0·5 in porous sphere	65	0·013	0·151	0·7045	0·7048	2·129	2·130
1·0 in porous sphere	20	0·013	0·149	0·7045	0·7047	2·129	2·130
0·02 in napthalene drops	99	—				0·6	2·2
0·38 in benzoic acid spheres	14	low				1210	
0·50 in benzoic acid spheres	13	high				1210	
0·75 in benzoic acid spheres	15	high				1210	
0·5 in benzoic, cinnamic acid and 2-napthol spheres	13	low				1200	
0·04 in water drops	78	0·02				1·2	
1·0 in napthalene spheroids	16	low		1·0126		2·5	
1·0–1·4 in porous spheres	56	0·03	0·23			0·6	1·7
0·8–6·1 in porous spheres	37	high				0·6	
0·2–0·5 in celite spheres	16	0·10					
1·4–2·0 in celite spheres		low					
0·5 in benzoic acid, napthalene and copper spheres	171	low		0·7	7·3	2·5	2100
0·5 in silver sphere	29	0·013	0·151	0·7045	0·7047		
1·0 in silver sphere	14	0·013	0·151	0·7046	0·7047		
0·87 in copper sphere	62	low		2·5	7		
4·0 in copper sphere	30	0·02	0·12	0·7			
1·5 in copper sphere	4	1		2·2	6·8		
6·0 in copper sphere	8	0·02		0·7			
9·0 in copper sphere	8	0·02		0·7			
12·0 in copper sphere	8	0·02		0·7			
0·28–0·50 in steel spheres	79			0·7	380		
0·125–0·625 in spheres	§	—	—	0·7		—	—
2·8–5·9 in nickel plated copper spheres	§	0·004	0·028	0·7		—	—
9·0 in copper sphere	§	0·02		0·7		—	—
5·0 in iron sphere	§	—	—	0·7		—	—
0·38–2·5 in steel and brass spheres	§	—	—	0·7		—	—
0·02–0·06 in spheres	§	—	—	—	—	127	1400
0·271 in sphere	§	low		—	—	0·5	2·7

† See nomenclature for definition of terms.
‡ Viscosity ratio defined as ν_∞/ν_i.
§ Data not used in evaluation of coefficients.

experimental conditions†

Viscosity ratio‡ min.	Viscosity ratio‡ max.	Range of data: Reynolds number min.	Reynolds number max.	Nusselt or Sherwood number min.	Nusselt or Sherwood number max.	Source	Remarks
1·34	1·36	66	322	6	14	[16]	air jet
		900	7200	15	73	[15, 24]	air jet
		18	7300	24	67	[25]	air jet
1·0		50	1000	17	30	[5]	air jet
		490	7500	150	430	[29]	water tunnel
		100	800	80	300	[35]	water tunnel
		100	700	70	320	[37]	water tunnel
		600	140000	36	360	[57]	water tunnel
1·0		0	200	2	9	[6]	air jet
1·0		200	6000	5	70	[48]	wind tunnel
1·157		3000	4000	40	400	[11]	wind tunnel
1·0		0	5000	0	50	[7]	wind tunnel
		1000	4000	30	60	[27]	wind tunnel
0·8		1000	50000	20	400	[29]	water tunnel
1·0		10	10000	9	265	[58]	water trough and tunnel
		900	7300	18	62	[13, 14, 25]	air jet
		1800	7500	24	64	[25]	air jet
1·0	2·7	1000	50000	20	400	[30]	water tunnel
0·936		1800	200000	90	700	[21]	wind tunnel
1·0	2·7	5	480000	0	1500	[46]	water tunnel
		87000	667000	300	800	[18]	wind tunnel
		130000	1000000	314	1000	[18]	wind tunnel
		177000	1200000	800	1500	[18]	wind tunnel
—	—	50	3000	4	18	[9]	glass tube
—	—	30000	300000	145	610	[12]	wind tunnel
—	—	130000	1000000	630	2000	[17]	wind tunnel
—	—	44000	150000	150	300	[20]	fan
—	—	10	100000	3	700	[23]	wind tunnel
—	—	4	100	4	57	[28]	water tunnel
0·7	1·0	800	5700	19	40	[10]	jet

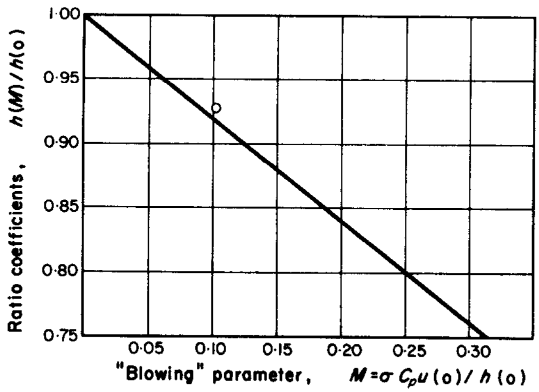

FIG. 7. Effect of "blowing parameter" upon material transport.

past the sphere is increased by the amount of reduction in tunnel area or "solid blockage". A first-order linear correction can be made, such as was carried out by Brown *et al.* [46]. Wake blockage effects were also brought out by Maskell [62]. In addition, there exists a slight alteration of the velocity profile as the free-stream turbulence is varied, emphasizing that corrections for local velocity can be made accurately only from a detailed knowledge of the actual velocity profile prevailing at the time of measurement.

RESULTS

Each of the numerous complicating and secondary factors discussed above contributes to the variation of the Frössling number. An expression that may be suitable for macroscopic thermal, thermal with simultaneous material, and material transport can be written for subcritical flow as:

$$Fs_{\infty}^{*} = A^{*}\left(\frac{v_{\infty}}{v_{i}}\right)^{0.16} + [B^{*}\alpha_{t}(\alpha_{t} + C^{*}) + D^{*}]\, Re_{\infty}^{\frac{1}{2}}\, Pr_{m,\infty}^{\frac{1}{3}} \quad (14)$$

and similarly for material transport, replacing the Prandtl number, *Pr*, with the Schmidt number, *Sc*. The range of conditions involved in the data used to determine the coefficients of equation (14) is presented in Table 1. A sample of the data employed constitutes Table 2. The entire background of the experimental information employed is available [1]. The expressions were utilized in describing the data of Sage and coworkers [13–15, 24, 25], and the coefficients are recorded in Table 3. The recommended coefficients and associated statistics and applicable conditions for the other data are indicated in Table 4. The values of the coefficients recorded in Tables 3 and 4 were obtained by conventional linear regression analysis.

The applicability of this relation for subcritical thermal and material transport is shown in Fig. 3, where data are given for both liquids and gases over the range of Prandtl numbers from 0·6 to 400, and Schmidt numbers from 0·6 to 1400. No gross deviations between the Frössling numbers for liquids and gases or between thermal and material transport have been encountered. To investigate systematic deviations from equation (14), it appears that experimental work will be required involving liquid metals where Lewis numbers deviate substantially from unity. Equation (14) is applicable for transport from an isothermal or isoconcentration surface, based on a local free-stream velocity immediately upstream of stagnation with no "blockage effects", for negligible "blowing velocity" and free convective and radiant transfer. The expression is limited to Lewis numbers near unity, which is typical for common fluids. The limitation in Reynolds number of equation (14) is shown in Fig. 3. The curves of Fig. 2 and the subcritical portions of the curves in Fig. 3 were obtained from equation (14).

For supercritical flows of Reynolds numbers above 360000, the following expression is applicable:

$$Fe_{\infty}^{*} = G^{*} + H^{*}\, Re_{\infty}^{\frac{1}{2}}\, Pr_{m,\infty}^{\frac{1}{3}} \quad (15)$$

and similarly for material transport, using the Schmidt number in place of the Prandtl number. The data employed to establish the coefficients of equation (15) are designated in Table 1, and the values of the coefficients in a part of Table 4.

Table 2. Sample of macroscopic transport from spheres†

1·0-in silver sphere§							
Test number	Air temperature (°F)	Turbulence level	Reynolds number	Interface temperature (°F)	Thermal flux (Btu/s)	Viscosity ratio‡	Nusselt number
176a	100·54	0·013	1880	160·03	$2·337 \times 10^{-3}$	0·8372	24·2
177	100·54	0·013	1884	160·03	1·632	0·8372	24·0
176b	99·98	0·013	3730	160·03	2·337	0·8363	34·2
175	100·02	0·013	7500	160·25	3·336	0·8363	48·6
174	100·02	0·013	7500	160·25	3·318	0·8363	48·6
185	100·15	0·051	7480	160·03	3·478	0·8367	51·1
186	100·00	0·054	3715	160·25	2·416	0·8359	35·4
184	100·13	0·064	7530	160·16	3·581	0·8363	52·5
183	99·81	0·067	3740	159·98	2·382	0·8358	34·9
181	100·09	0·086	7430	160·26	3·602	0·8355	52·7
182	100·09	0·091	3720	160·17	2·466	0·8357	36·2
180	100·19	0·132	7490	160·30	4·348	0·8364	63·8
179	100·04	0·143	3720	160·19	2·858	0·8355	41·9
178	100·11	0·151	1880	160·02	1·856	0·8367	27·3

† See nomenclature for definition of terms.
‡ Viscosity ratio defined as ν_∞/ν_i.
§ The Prandtl number used in the calculations varied between 0·7046 and 0·7047.

Table 3. Coefficients of equation (14) *for small spheres*

Description of sphere	Number of data points		Coefficients				Deviation, fraction	
	used	rejected†	*A*	*B*	*C*	*D*	average‡	standard§
thermal								
0·5 in silver sphere	30	0	0·5445	0·08206	0·03970	0·001579	0·0000475	0·0105
1·0 in silver sphere	14	0	0·5270	0·1548	−0·02429	0·001229	0·000128	0·0173
thermal and material								
0·5 in porous sphere	57	7	0·4748	0·2357	−0·03464	0·001909	0·000295	0·0255
1·0 in porous sphere	20	0	0·5401	0·1290	0·005893	0·001182	0·000121	0·0141
material								
0·5 in porous sphere	56	7	0·5037	0·06256	0·06240	0·0002396	0·000104	0·0188

† Point statistically rejected when deviation exceeds 5σ.
‡ Average deviation defined by:

$$s = [\sum_1^{N_p} \{w[(Fs^*_{\infty,e} - Fs^*_{\infty,c})/Fs^*_{\infty,e}]\}]/N_p.$$

§ Standard deviation defined by:

$$\sigma = [\sum_1^{N_p} \{w[(Fs^*_{\infty,e} - Fs^*_{\infty,c})/Fs^*_{\infty,e}]^2\}/(N_p - N_c)]^{\frac{1}{2}}.$$

Table 4. Recommended

	Subcritical flow‡			
	Nature of			
	Thermal		Thermal and material	
	min.	max.	min.	max.
Applicable range				
Sphere diameter (in)	0·25	12·0	0·02	2·0
Reynolds number	1·2	300000	2·1	119500
Turbulence level, fraction	0·005	0·15	0·01	0·15
Prandtl or Schmidt number	0·70	380·0	0·697	0·721
Viscosity ratio	0·66	2·7	1·00	1·36
Nusselt or Sherwood number	6·9	543·0	2·89	78·6
Number of data points				
Used	388		143	
Rejected‖	5		3	
Recommended coefficients				
A^* equation (14)	0·5747		0·5604	
B^* equation (14)	0·1674		0·3988	
C^* equation (14)	−0·05628		−0·1110	
D^* equation (14)	0·001449		0·002073	
G^* equation (15)				
H^* equation (15)				
Deviation, fraction				
Average¶	0·00652		0·00162	
Standard††	0·0786		0·0543	

† See nomenclature for definition of terms.

‡ Subcritical flow, equation (14).

$$Fs^*_\infty = A^*(v_\infty/v_i)^{0.16} + [B^*\alpha_t(\alpha_t + C^*) + D^*]\, Re^{\frac{1}{2}}_\infty\, (Pr^{\frac{1}{3}}_{m,\infty} \text{ or } Sc^{\frac{1}{3}}_{m,\infty}).$$

§ Supercritical flow, equation (15)

$$Fs^*_\infty = G^* + H^*\, Re^{\frac{1}{2}}_\infty\, Pr^{\frac{1}{3}}_{m,\infty}.$$

coefficients for equation (14) *or* (15)†

				Supercritical flow§	
transport					
Material		Combined		Thermal	
min.	max.	min.	max.	min.	max.
0·02	2·0	0·02	12·0	4·0	12·0
2·1	48 300	1·2	300 000	250 000	1 330 000
0·005	0·23	0·005	0·23	0·005	0·07
0·593	1210·0	0·593	1210·0	0·708	
1·00	1·37	0·66	2·7	1·00	
2·58	198·0	2·89	543·0	550·0	1418·0
436		947		22	
4		11		2	
0·5454		0·5483			
0·09904		0·1212			
−0·0004096		−0·04595			
0·001362		0·001656			
				1·171	
				0·000159	
0·00693		0·00630		0·0102	
0·102		0·0881		0·0477	

‖ Points statistically rejected when deviation exceeds 5σ.

¶ Average deviation defined by:

$$s = \{\sum_{1}^{N_p} w\,[(Fs^*_{\infty,e} - Fs^*_{\infty,c})/Fs^*_{\infty,e}]\}/N_p.$$

†† Standard deviation defined by:

$$\sigma = [\{\sum_{2}^{N_p} w\,[(Fs^*_{\infty,e} - Fs^*_{\infty,c})/Fs^*_{\infty,c}]\}^2/(N_p - N_c)]^{\frac{1}{2}}.$$

The applicability of equation (15) for supercritical flows is shown in Fig. 3. Although this relation is based on a much more limited range of Prandtl and Schmidt numbers, no gross deviations were encountered. Frössling numbers for the water tunnel measurements of Brown [46] appear slightly higher than those for the wind tunnel measurements of Wadsworth [21] involving air. Some of this behavior may be due to the deviation of the Lewis number from unity in the turbulent layers.

CONCLUSIONS

The intensity of free-stream turbulence markedly influences the transfer in the forward hemisphere and thus has a substantial effect on the macroscopic transport which increases with the Reynolds number, and such effects are well represented by the use of a Frössling number. The transport in the turbulent portion of the wake exhibits a higher dependence on the Reynolds number than the over-all transport, but is nearly independent of the level of turbulence. The transition Reynolds number for thermal transfer agrees with the behavior observed from drag measurements. Well above the transition Reynolds number, the effects of free-stream turbulence upon thermal transport are negligible.

With all other fluid and flow properties taken at free-stream conditions, the effects of variable molecular properties for the available data can roughly be described by a ratio of the kinematic viscosity of the free stream to that of the interface raised to a power. The "blowing velocity" associated with material transfer results in a decrease in the transfer coefficient by about 7 per cent for most measurements upon spheres.

ACKNOWLEDGEMENT

Virginia Berry assisted with the calculations reported, while Florence Nash contributed to the assembly of the manuscript and June Gray to the preparation of the figures. The senior author was the recipient of the Peter E. Fluor Memorial Fellowship.

REFERENCES

1. T. R. GALLOWAY and B. H. SAGE, Document 9428 Am. Doc. Inst., Washington 25, D.C. (1967).
2. H. W. MORSE, On evaporation from the surface of a solid sphere, *Proc. Am. Acad. Arts Sci.* **45**, 363 (1910).
3. I. LANGMUIR, The evaporation of small spheres, *Phys. Rev.* **12**, 368 (1918).
4. N. FUCHS, Concerning the velocity of evaporation of small droplets in a gas atmosphere, *Phys. Z. SowjUn* **6**, 224 (1934); translated NACA, TM 1160.
5. N. FRÖSSLING, Über die Verdunstung Fallender Tropfen, *Beitr. Geophys.* **52** (1/2), 170 (1938).
6. W. E. RANZ and W. R. MARSHALL, Evaporation from drops, *Chem. Engng Progr.* **48**, Part I, 141; Part II, 173 (1952).
7. R. W. POWELL, Further experiments on the evaporation of water from saturated surfaces, *Trans. Instn Chem. Engrs* **18**, 36 (1940).
8. H. KRAMERS, Heat transfer from spheres to flowing media, *Physica, 's Grav.* **12**, 61 (1946).
9. Y. S. TANG, J. M. DUNCAN and H. E. SCHWEYER, Heat and momentum transfer between a spherical particle and air streams, NACA TN 2867 (1951).
10. R. D. INGEBO, Vaporization rates and heat-transfer coefficients for pure liquid drops, NACA TN 2368 (1951).
11. D. S. MAISEL and T. K. SHERWOOD, Evaporation of liquids into turbulent gas streams, *Chem. Engng Progr.* **46**, 131 (1950).
12. L. LOITZIANSKI and B. SCHWAB, The thermal scale of turbulence, *Tech. Phys. U.S.S.R.* **2** (5) (1935).
13. W. W. SHORT, R. A. S. BROWN and B. H. SAGE, Thermal transfer in turbulent gas streams. Effect of turbulence on local transport from spheres, *J. Appl. Mech.* **27E**, 393 (1960).
14. K. SATŌ and B. H. SAGE, Thermal transfer in turbulent gas streams—effect of turbulence on macroscopic transport from spheres, *Trans. Am. Soc. Mech. Engrs* **80**, 1380 (1958).
15. R. A. S. BROWN, K. SATŌ and B. H. SAGE, Material transfer in turbulent gas streams. Effect of turbulence on macroscopic transport from spheres, *Chem. Engng Data Ser.* **3**, 263 (1958).
16. N. T. HSU, K. SATŌ and B. H. SAGE, Material transfer in turbulent gas streams, *Ind. Engng Chem.* **46**, 870 (1954).
17. L. G. LAUTMAN and W. C. DROEGE, Thermal conductances about a sphere subjected to forced convection, Air Materiel Command, Serial No. AIRL A6118 50-15-3 (1950).
18. G. XENAKIS, A. E. AMERMAN and R. W. MICHELSON, An investigation of the heat-transfer characteristics of spheres in forced convection, Wright Air Development Center, Ohio, WADC Tech. Report 53-117 (1953).
19. A. E. AMERMAN, An investigation of the heat-transfer characteristics of spheres in forced convection, Wright Air Development Center, Ohio, WADC Tech. Report 53-117, Supplement 1 (1953).
20. J. R. CARY, The determination of local forced-con-

vection coefficients for spheres, *Trans. Am. Soc. Mech. Engrs* **75**, 483 (1953).
21. J. Wadsworth, The experimental examination of the local heat transfer on the surface of a sphere when subjected to forced convective cooling, Nat. Res. Council Can., Div. Mech. Eng., Report No. MT-39 (1958).
22. L. S. Klyachko, Heat transfer between a gas and a spherical surface with the combined action of free and forced convection, *J. Heat Transfer* **85C**, 355 (1963).
23. T. Yuge, Experiments on heat transfer from spheres including combined natural and forced convection, *J. Heat Transfer* **82C**, 214 (1960).
24. R. A. S. Brown and B. H. Sage, Material transfer in turbulent gas streams, *J. Chem. Engng Data* **6**, 355 (1961).
25. E. Venezian, M. J. Crespo and B. H. Sage, Thermal and material transfer in turbulent gas streams: one-inch spheres, *A.I.Ch.E. Jl* **8**, 383 (1962).
26. L. B. Torobin and W. H. Gauvin, Fundamental aspects of solid-gas flow, *Can. J. Chem. Engng* **37**, 167 (1959).
27. I. S. Pasternak and W. H. Gauvin, Turbulent heat and mass transfer from stationary particles, *Can. J. Chem. Engng* **38**, 35 (1960).
28. R. M. Griffith, Mass transfer from drops and bubbles, *Chem. Engng Sci.* **12**, 198 (1960).
29. M. Linton and K. L. Sutherland, Transfer from a sphere into a fluid in laminar flow, *Chem. Engng Sci.* **12**, 214 (1960).
30. G. C. Vliet and G. Leppert, Forced convection heat transfer from an isothermal sphere to water, *J. Heat Transfer* **83C**, 163 (1961).
31. S. Evnochides and G. Thodos, Simultaneous mass and heat transfer in the flow of gases past single spheres, *A.I.Ch.E. Jl* **7**, 78, (1961).
32. A. Gupta and G. Thodos, Mass and heat transfer in the flow of fluids through fixed and fluidized beds of spherical particles, *A.I.Ch.E. Jl* **8**, 608 (1962).
33. J. T. L. McConnachie and G. Thodos, Transfer processes in the flow of gases through packed and distended beds of spheres, *A.I.Ch.E. Jl* **9**, 60 (1963).
34. R. B. Keey and J. B. Glen, Mass transfer from solid spheres, *Can. J. Chem. Engng* **42**, 227 (1964).
35. F. H. Garner and R. W. Grafton, Mass transfer in fluid flow from a solid sphere, *Proc. R. Soc.* **224**, 64 (1954).
36. F. H. Garner and R. B. Keey, Mass-transfer from single solid spheres—II, *Chem. Engng Sci.* **9**, 218 (1959).
37. F. H. Garner and R. B. Keey, Mass-transfer from single solid spheres—I, *Chem. Engng Sci.* **9**, 119 (1958).
38. F. H. Garner and R. D. Suckling, Mass transfer from a soluble solid sphere, *A.I.Ch.E. Jl* **4**, 114 (1958).
39. A. M. Kuethe, W. W. Willmarth and G. H. Crocker, Turbulence field near the stagnation point on blunt bodies of revolution, *Proc. Heat Transf. Fluid Mech. Inst.* p. 10. Stanford University Press (1961).
40. P. Harriott, A random eddy modification of the penetration theory, *Chem. Engng Sci.* **17**, 149 (1962).
41. A. R. Büyüktür, J. Kestin and P. F. Maeder, Influence of combined pressure gradient and turbulence on the transfer of heat from a plate, *Int. J. Heat Mass Transfer* **7**, 1175 (1964).
42. J. Kestin and P. F. Maeder, Influence of turbulence on transfer of heat from cylinders, NACA TN 4018 (1957).
43. R. A. Seban, The influence of free stream turbulence on the local heat transfer from cylinders, *J. Heat Transfer* **82C**, 101 (1960).
44. E. W. Comings, J. T. Clapp and J. F. Taylor, Air turbulence and transfer processes, *Ind. Engng Chem.* **40**, 1076 (1948).
45. W. H. Giedt, Effect of turbulence level of incident air stream on local heat transfer and skin friction on a cylinder, *J. Aeronaut. Sci.* **18**, 725 (1951).
46. W. S. Brown, C. C. Pitts and G. Leppert, Forced convection heat transfer from a uniformly heated sphere, *J. Heat Transfer* **84C**, 133 (1962).
47. C. C. Lin, *Turbulent Flows and Heat Transfer*, p. 529. Princeton University Press (1959).
48. A. H. P. Skelland and A. R. H. Cornish, Mass transfer from spheroids to an air stream, *A.I.Ch.E. Jl* **9**, 73 (1963).
49. R. W. Grafton, Prediction of mass transfer from spheres and cylinders in forced convection, *Chem. Engng Sci.* **18**, 457 (1963).
50. P. D. Richardson, Heat and mass transfer in turbulent separate flows, *Chem. Engng Sci.* **18**, 149 (1963).
51. P. D. Richardson, Estimation of the heat transfer from the rear of an immersed body to the region of separated flow, AD-290339, ARL 62-423 (1962).
52. G. E. Kinard, F. S. Manning and W. P. Manning, A new correlation for mass transfer from single spheres, *Br. Chem. Engng* **8** (5), 326 (1963).
53. M. H. I. Baird and A. E. Hamielec, Forced convection transfer around spheres at intermediate Reynolds numbers, *Can. J. Chem. Engng* **40**, 119 (1962).
54. A. E. Hamielec, S. H. Storey and J. M. Whitehead, Viscous flow around fluid spheres at intermediate Reynolds numbers (II), *Can. J. Chem. Engng* **41**, 246 (1963).
55. A. C. Lochiel and P. H. Calderbank, Mass transfer in the continuous phase around axisymmetric bodies of revolution, *Chem. Engng Sci.* **19**, 471 (1964).
56. E. Ruckenstein, On mass transfer in the continuous phase from spherical bubbles or drops, *Chem. Engng Sci.* **19**, 131 (1964).
57. L. R. Steele and C. J. Geankoplis, Mass transfer from a solid sphere to water in highly turbulent flow, *A.I.Ch.E. Jl* **5**, 178 (1959).
58. P. N. Rowe, K. T. Claxton and J. B. Lewis, Heat and mass transfer from a single sphere in an extensive flowing fluid, *Trans. Instn Chem Engrs* **43**, 14 (1965).
59. B. G. van der Hegge Zijnen, Heat transfer from horizontal cylinders to a turbulent air flow, *Appl. Scient. Res.* **7A**, 205 (1957–8).
60. W. W. Short, Heat transfer and sublimation at a stagnation point in potential flow, *J. Appl. Mech.* **27**, 613 (1960).
61. B. H. Sage and P. A. Longwell, Heat transfer and sublimation at a stagnation point in potential flow, *J. Appl. Mech.* **28**, 471 (1961).

62. E. C. MASKELL, A theory of the blockage effects on bluff bodies and stalled wings in a closed wind tunnel, Royal Aircraft Establishment. Report No. Aero 2685, (November 1963).
63. I. KOROBKIN, Discussion of local laminar heat transfer coefficients for spheres and cylinders in incompressible flow, A.S.M.E. Paper 54-F-18 (1954).
64. T. R. GALLOWAY and B. H. SAGE, Thermal and material transfer in turbulent gas streams. A method of prediction for spheres, *Int. J. Heat Mass Transfer* 7, 283 (1964).
65. H. SCHLICTING, *Boundary Layer Theory*. McGraw-Hill, New York (1960).
66. T. R. GALLOWAY and B. H. SAGE, Thermal and material transfer from spheres. Prediction of local transport, *Int. J. Heat Mass Transfer*, To be published.
67. H. L. DRYDEN, G. B. SCHUBAUER, W. C. MOCK JR. and H. K. SKRAMSTAD, NACA Report No. 581 (1937).

A more detailed form of this paper (or extended version, or material supplementary to this article) has been deposited as Document number 9428 with the ADI Auxiliary Publications Project, Photoduplication Service, Library of Congress, Washington, D.C. 20540. A copy may be secured by citing the Document number and by remitting $2.50 for photoprints, or $1.75 for 35 mm microfilm. Advance payment is required. Make checks or money orders payable to: Chief, Photoduplication Service, Library of Congress.

Résumé— Une corrélation pour le nombre de Frössling dans le cas du transport macroscopique ou moyen dans l'espace à partir des sphères a été préparée à partir des renseignements disponibles concernant l'effet des propriétés moléculaires du fluide, des conditions de l'écoulement, et du niveau de turbulence. Ce dernier est le plus important pour un nombre de Reynolds de 40000 et devient relativement négligeable pour des nombres de Reynolds superieurs à 250000. Des expressions analytiques, de nature empirique, ont été établies pour décrire l'influence du nombre de Reynolds, du niveau de turbulence et des propriétés moléculaires du fluide sur le nombre de Frössling. La précision dans la prévision du nombre de Frössling est probablement voisine de la précision des données expérimentales. On a trouvé utile l'emploi du rapport de la viscosité cinématique de l'écoulement libre à celle à l'interface élevé à une certaine puissance pour corriger les changements des propriétés moléculaires à travers la couche limite, bien que cette méthode soit empirique. Les résultats sont présentés sous forme de courbes et de tableaux.

Zusammenfassung—Aus einer Durchsicht verfügbarer Information über den Einfluss molekularer Eigenschaften einer Flüssigkeit, des Strömungsverhaltens und des Turbulenzgrades wurde für den makroskopischen oder durchschnittlichen Raumtransport von Kugeln eine Korrelation aufgrund des Frössling-Zahl ausgearbeitet. Der Einfluss des Turbulenzgrades ist am ausgeprägtesten bei Reynolds–Zahlen von 40000 und wird relativ unbedeutend bei Reynolds-Zahlen über 250000. Analytische Ausdrücke, ihrer Natur nach aber empirische, wurden entwickelt, um den Einfluss der Reynolds-Zahl, des Turbulenzgrades und der molekularen Eigenschaften der Flüssigkeit auf die Frössling-Zahl zu beschreiben. Die Genauigkeit der Bestimmung der Frössling-Zahl ist wahrscheinlich vergleichbar mit jener der Versuchsdaten. Die Verwendung des Verhältnisses der kinematischen Zähigkeit des Freistromes zu jener der Zwischenschicht, erhoben in eine Potenz, zeigte sich als eine nützliche, wenn auch empirische Methode, Änderungen in den Molekulareigenschaften innerhalb der Grenzströmungen zu korrigieren. Die Ergebnisse sind in Diagrammen und Tabellen wiedergegeben.

Аннотация—Из обзора имеющихся данных относительно влияния молекулярных свойств жидкости, условий течения и степени турбулентности получено соотношение для числа Фрёсслинга для макроскопического или пространственно-среднего переноса. Влияние степени турбулентности больше всего заметно при числе Рейнолдьса 40000 и становится довольно незначительным при числах Рейнольдса выше 250000. Были получены аналитические выражения, эмпирические по своему характеру, для описания влияния числа Рейнолдьса, степени турбулентности и молекулярных свойств жидкости на число Фрёсслинга. Точность расчёта числа Фрёсслинга сравнима с точностью экспериментальных данных. Установлено, что отношение кинематической вязкости свободного потока к кинематической вязкости на поверхности раздела, возведенной в степень, является полезным, хотя и эмпирическим, методом учета изменения молекулярных свойств в пограничных течениях. Результаты представлены в виде графиков и таблиц.